Roll Divergence of a Canard-Controlled Missile with a Freely Spinning Tail

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An analytical study of the rolling motion of a missile with a freely spinning tail and canard control surfaces actuated by constant imposed hinge moments is presented. The study determines the equilibrium states of motion, investigates small perturbations, derives explicit conditions for stability, and examines large perturbations using numerical solutions of the full equations of motion. It is found that a divergence of the rolling motion occurs over a wide range of parameters.

Introduction

THE concept of a freely spinning tail, which eliminates or reduces the unwanted induced rolling moments of a missile, has drawn considerable attention in recent years.[1,2] The finned tail is connected to the missile body through a bearing, which transmits pitching and yawing moments but does not transmit rolling moments from the tail.

For missiles designed to have high maneuvering capabilities, it was proposed recently to use flight control methods that monitor lift forces or hinge moments of the control surfaces rather than their deflection angles. This should reduce the cross-coupling aerodynamic moments generated by cruciform control surfaces at high angles of attack.[3] Moreover, the pitching and yawing moments produced by control surfaces are, in general, nonmonotonic functions of the control deflection angle at high angles of attack, but are fairly linear functions of the hinge moment. Thus control by imposing deflection angles would not be practicable, and control by hinge-moment commands should be preferred.

The present study is concerned with the rolling motion of a missile in which the two proposed concepts are incorporated. The missile has a freely spinning tail, and its rolling motion is controlled by four canard surfaces on which monitored hinge moments are imposed. Particular attention is given to the fundamental case of zero hinge moments, i.e., free controls.

It should be expected at the outset that a missile with a freely spinning tail and free controls will have low damping in roll, positive or negative. In fact, free control surfaces tend to align themselves in the direction of the local airflow so that they will not produce strong roll-damping moments. Therefore, it is of interest to analyze the rolling motion and to determine its stability conditions.

The motion consists of missile roll and rotation of control surfaces about their hinges. To simplify the analysis, we assume 1) a pure roll, without pitching or yawing, at zero angle of attack; 2) a perfect bearing between the missile and its finned tail, fully preventing transfer of rolling moments; 3) a constant flight speed; and 4) identical cruciform control surfaces having the same imposed hinge moments and moving in unison. The equations of motion take into account aerodynamic and inertial cross couplings between the missile and the control surfaces. The aerodynamic terms include rolling-moment and hinge-moment derivatives with respect to the roll rate, control-surface deflection, and deflection rate. The inertial cross couplings include the hinge moment due to centrifugal force on the control surface, the hinge moment due to missile roll acceleration, and the rolling moment due to angular acceleration of control surfaces about their hinges.

The study determines the equilibrium states of motion, investigates analytically small perturbations for constant imposed hinge moments, and derives explicit conditions of stability. To examine the behavior of large perturbations, and the effects of missile freedom in pitch and yaw, numerical solutions are obtained for the case of pure roll and for the general case of motion in six degrees of freedom with nonunison initial deflections of the four control surfaces.

Equations of Motion

The state variables in the pure rolling motion are the missile rate of roll p and the angular deflection δ of the control surfaces about their hinges (see Fig. 1). It is assumed that the same control hinge moment H_c is applied at the four control surfaces, so that each surface has the same deflection angle.

In formulating the equations of motion, we use a Cartesian system (x_B,y_B,z_B) attached to the missile body, and a system (x,y,z) attached to a control surface (Fig. 1). The x_B axis is the missile roll axis directed forward, the z_B and z axes are taken along the hinge of a control surface, and the x axis coincides with x_B when $\delta=0$.

The position vector of the mass center of the control surface is

$$r_c = x_c(\cos\delta)\boldsymbol{i}_B - x_c(\sin\delta)\boldsymbol{j}_B + z_c\boldsymbol{k}_B \tag{1}$$

where x_c and z_c are the constant distances of the mass center from the hinge and (x_B,y_B) plane, respectively, and $\boldsymbol{i}_B,\boldsymbol{j}_B,\boldsymbol{k}_B$ are unit vectors along the (x_B,y_B,z_B) axes. The time derivatives of the unit vectors are

$$\frac{\mathrm{d}\boldsymbol{j}_B}{\mathrm{d}t} = p\boldsymbol{k}_B, \quad \frac{\mathrm{d}\boldsymbol{k}_B}{\mathrm{d}t} = -p\boldsymbol{j}_B, \quad \frac{\mathrm{d}\boldsymbol{i}_B}{\mathrm{d}t} = 0 \tag{2}$$

and the angular velocity vector of the control surface is

$$\omega \neq p\boldsymbol{i}_B - \frac{\mathrm{d}\delta}{\mathrm{d}t}\boldsymbol{k}_B \tag{3}$$

The equations of motion of a control surface[4] are then

$$m_c\frac{\mathrm{d}^2 \boldsymbol{r}_c}{\mathrm{d}t^2} = \boldsymbol{F} \tag{4}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}[I_c \cdot \omega] = \boldsymbol{M} \tag{5}$$

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where m_c and I_c are the mass and the inertia tensor of the control surface, respectively, F the force acting on the surface, and M the moment about the center of mass. Vectors F and M include aerodynamic forces and moments, reactions transmitted to the control surface through its hinge, and a force and moment imposed by the control system. Since a hinge cannot transmit moments around itself, the reactions consist of three force and two moment components. A single equation is obtained[5] by eliminating the five reaction terms from the six scalar equations arising from Eqs. (4) and (5). For moderate deflection angles we may simplify the expressions by taking

$$\cos\delta = 1, \qquad \sin\delta = \delta \tag{6}$$

The resulting equation of motion of a control surface is[5]

$$I_h\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} - I_h p^2\delta + I_{xz}\frac{\mathrm{d}p}{\mathrm{d}t} - H_a = H_c \tag{7}$$

where I_h is the moment of inertia of the control surface about its hinge, I_{xz} its inertia cross product with respect to the hinge and the x axis, H_c the hinge moment imposed by the control system, and H_a the aerodynamic hinge moment. Within the range of linear aerodynamics we have

$$H_a = \tfrac{1}{2}\rho V^2 Sc\left(C_{h\delta}\delta + C_{hp}p + C_{h\dot\delta}\frac{\mathrm{d}\delta}{\mathrm{d}t}\right) \tag{8}$$

where S and c are the area and reference chord of a control surface, ρ the air density, and V the flight speed. In Eq. (7), $I_h P^2\delta$ is due to the centrifugal forces acting on the control surface, and $I_{xz}(\mathrm{d}p/\mathrm{d}t)$ represents the hinge moment due to roll acceleration.

The rolling motion of the missile body is governed by the equation[4]

$$I_B\frac{\mathrm{d}p}{\mathrm{d}t} = M_B \tag{9}$$

where I_B is the moment of inertia of the body about the roll axis, and M_B the rolling moment transmitted to the body through the four hinges. M_B involves the reaction forces and moments that can be eliminated using Eqs. (4) and (5). The ensuing equation of motion in roll is

$$I_R\frac{\mathrm{d}p}{\mathrm{d}t} + 4I_{xz}\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} + 8I_h p\delta\frac{\mathrm{d}\delta}{\mathrm{d}t} - 4L_a = 0 \tag{10}$$

Here I_R denotes the moment of inertia of the missile body with the four control surfaces (at $\delta = 0$) about the roll axis, and L_a is the aerodynamic rolling moment generated by a control surface,

$$L_a = \tfrac{1}{2}\rho V^2 Sc\left(C_{l\delta}\delta + C_{lp}p + C_{l\dot\delta}\frac{\mathrm{d}\delta}{\mathrm{d}t}\right) \tag{11}$$

The signs of the aerodynamic derivatives are normally

$$C_{l\delta} > 0, \quad C_{lp} < 0, \quad C_{h\delta} < 0, \quad C_{hp} > 0, \quad C_{h\dot\delta} < 0 \tag{12}$$

To make the equations nondimensional, we define a reference time τ by

$$\tau^2 = \frac{2I_R}{\rho V^2 Sc} \tag{13}$$

and denote

$$\begin{aligned} \bar t &= t/\tau & \bar p &= \tau p \\ \bar C_{hp} &= C_{hp}/\tau & \bar C_{lp} &= C_{lp}/\tau \\ \bar C_{h\dot\delta} &= C_{h\dot\delta}/\tau & \bar C_{l\dot\delta} &= C_{l\dot\delta}/\tau \\ J_h &= I_h/I_R & J_{xz} &= I_{xz}/I_R \\ C_{hc} &= H_c/(\tfrac{1}{2}\rho V^2 Sc) \end{aligned} \tag{14}$$

The equations of motion expressed in nondimensional form are

$$J_h\frac{\mathrm{d}^2\delta}{\mathrm{d}\bar t^2} - J_h\bar p^2\delta + J_{xz}\frac{\mathrm{d}\bar p}{\mathrm{d}\bar t} - C_{h\delta}\delta - \bar C_{hp}\bar p - \bar C_{h\dot\delta}\frac{\mathrm{d}\delta}{\mathrm{d}\bar t} = C_{hc} \tag{15}$$

$$\frac{1}{4}\frac{\mathrm{d}\bar p}{\mathrm{d}\bar t} + J_{xz}\frac{\mathrm{d}^2\delta}{\mathrm{d}\bar t^2} + 2J_h\bar p\delta\frac{\mathrm{d}\delta}{\mathrm{d}\bar t} - C_{l\delta}\delta - \bar C_{lp}\bar p - \bar C_{l\dot\delta}\frac{\mathrm{d}\delta}{\mathrm{d}\bar t} = 0 \tag{16}$$

Equilibrium Motions

The equilibrium states of roll are found by setting

$$\bar p - \bar p_e, \qquad \frac{\mathrm{d}\bar p_e}{\mathrm{d}\bar t} = 0 \qquad \delta = \delta_e, \qquad \frac{\mathrm{d}\delta_e}{\mathrm{d}\bar t} = 0 \tag{17}$$

The equations of motion [Eqs. (15) and (16)] then give

$$\bar p_e = -\frac{C_{l\delta}}{\bar C_{lp}}\delta_e \tag{18}$$

$$\left(C_{h\delta} - \frac{\bar C_{hp}}{\bar C_{lp}}C_{l\delta}\right)\delta_e + J_h\left(\frac{C_{l\delta}}{\bar C_{lp}}\right)^2\delta_e^3 = -C_{hc} \tag{19}$$

which determine the control-surface deflection δ_e and missile roll rate p_e in an equilibrium state with a given control hinge-moment coefficient C_{hc}. The cubic term in Eq. (19) cannot be neglected in the case of a freely spinning tail, since the coefficient of the linear term is small relative to that of the cubic term.

We will denote

$$\Delta = -C_{h\delta} + (\bar C_{hp}/\bar C_{lp})C_{l\delta} \tag{20}$$

The number of real roots of Eq. (19) depends upon the sign of

$$Q = -\Delta^3 + (27/4)J_h(C_{l\delta}/\bar C_{lp})^2 C_{hc}^2 \tag{21}$$

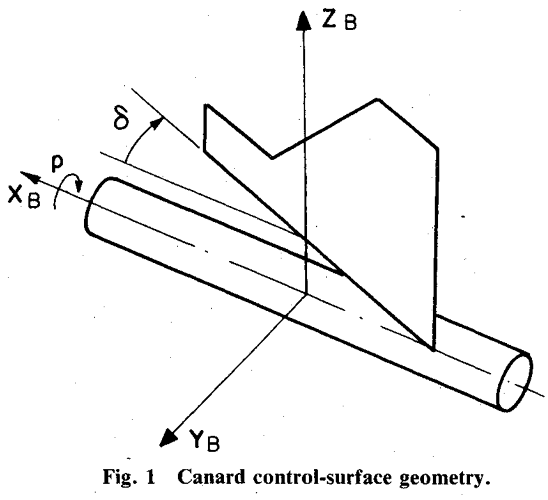

Fig. 1 Canard control-surface geometry.

The motion has three equilibrium states (real roots) when $Q<0$, and one equilibrium state when $Q>0$. Both cases can occur since normally $C_{h\delta}<0$. Figure 2 shows the equilibrium roll rate as a function of the imposed hinge-moment coefficient C_{hc} and the aerodynamic hinge-moment derivative $C_{h\delta}$ for plausible values of the remaining parameters.

When the control surfaces are free ($C_{hc}=0$) the equilibrium states are the state of rest

$$\bar{p}_e=0, \qquad \delta_e=0$$

and the steady roll

$$\bar{p}_e=\pm\sqrt{\Delta/J_h}, \qquad \delta_e=\mp(\bar{C}_{lp}/C_{l\delta})\sqrt{\Delta/J_h} \tag{22}$$

provided $\Delta>0$.

A control derivative of equilibrium motions can be defined as

$$\frac{\mathrm{d}C_{hc}}{\mathrm{d}\delta_e}=\Delta-3J_h\bar{p}_e^2 \tag{23}$$

When this derivative has a negative sign, decreasing the hinge moment will produce larger control-surface deflections and higher roll rates. Therefore, a necessary, but insufficient, condition for stability of the rolling motion is

$$\frac{\mathrm{d}C_{hc}}{\mathrm{d}\delta_e}>0 \tag{24}$$

For small equilibrium roll rates, the control derivative is determined mainly by the aerodynamic parameter Δ. At large rates of roll the quadratic term in Eq. (23) will be dominant, the control derivative will be negative, and the rolling motion will diverge. The destabilizing quadratic term, as well as the corresponding cubic term of Eq. (19), is due to the hinge moment of the centrifugal forces on the control surface.

The parameter Δ, defined in Eq. (20), expresses the effective aerodynamic stability of a control surface about its hinge, taking into account the aerodynamic coupling with the rolling motion. Moreover, the parameter Δ is related to the effective damping in the roll of the missile, taking into account the floating of the control surfaces,

$$\Delta=-\frac{C_{h\delta}}{\bar{C}_{lp}}\left(\bar{C}_{lp}-\frac{C_{l\delta}}{C_{h\delta}}\bar{C}_{hp}\right) \tag{25}$$

Elementary considerations show that for a missile with a freely spinning tail the term $C_{l\delta}\,\bar{C}_{hp}/C_{h\delta}$ of Δ is similar in magnitude to the term $\bar{C}_{lp}$, since the roll-damping moment C_{lp} is produced by control surfaces only. Thus the parameter Δ is relatively small.

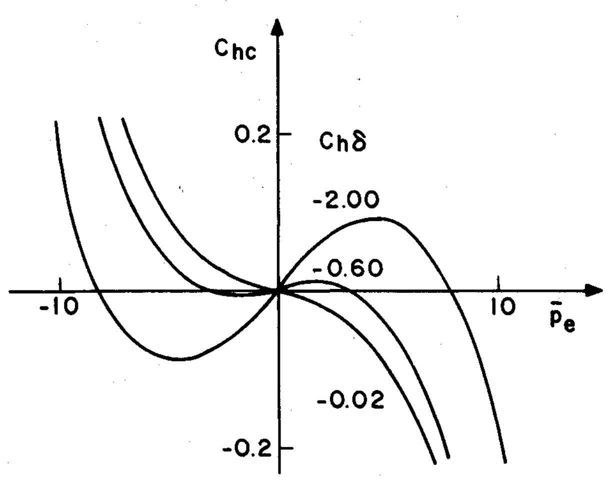

Fig. 2 Equilibrium roll rate.

Stability Conditions

To investigate the stability of the rolling motion more closely, we consider small deviations from an equilibrium state for constant-control hinge moments, and set

$$\delta=\delta_e+\delta^*, \quad \bar{p}=\bar{p}_e+p^*, \quad C_{hc}=\text{const} \tag{26}$$

where p^* and δ^* are small perturbations. Linearizing the equations of motion [Eqs. (15) and (16)] gives

$$J_h\frac{\mathrm{d}^2\delta^*}{\mathrm{d}\bar{t}^2}+J_{xz}\frac{\mathrm{d}p^*}{\mathrm{d}\bar{t}}-\bar{C}_{h\dot{\delta}}\frac{\mathrm{d}\delta^*}{\mathrm{d}\bar{t}}-\hat{C}_{h\delta}\delta^*-\hat{C}_{hp}p^*=0 \tag{27}$$

$$\frac{1}{4}\frac{\mathrm{d}p^*}{\mathrm{d}\bar{t}}+J_{xz}\frac{\mathrm{d}^2\delta^*}{\mathrm{d}\bar{t}^2}-\hat{C}_{l\dot{\delta}}\frac{\mathrm{d}\delta^*}{\mathrm{d}\bar{t}}-C_{l\delta}\delta^*-\bar{C}_{lp}p^*=0 \tag{28}$$

where we denote for brevity

$$\hat{C}_{h\delta}=C_{h\delta}+J_h\bar{p}_e^2$$
$$\hat{C}_{hp}=\bar{C}_{hp}+2J_h\bar{p}_e\delta_e$$
$$\hat{C}_{l\dot{\delta}}=2J_h\bar{p}_e\gamma_e \tag{29}$$

The solutions have the form

$$\delta^*=ae^{\lambda\bar{t}}, \qquad p^*=be^{\lambda\bar{t}} \tag{30}$$

and the resulting characteristic equation is cubic,[6]

$$A\lambda^3+B\lambda^2+C\lambda+D=0 \tag{31}$$

Here

$$A=\tfrac{1}{4}J_h-J_{xz}^2$$
$$B=-J_h\bar{C}_{lp}-\tfrac{1}{4}\bar{C}_{h\dot{\delta}}+J_{xz}(\hat{C}_{hp}+\hat{C}_{l\dot{\delta}})$$
$$C=-\tfrac{1}{4}\hat{C}_{h\delta}+J_{xz}C_{l\delta}+\bar{C}_{h\dot{\delta}}\bar{C}_{lp}-\hat{C}_{l\dot{\delta}}\hat{C}_{hp}$$
$$D=\hat{C}_{h\delta}\bar{C}_{lp}-C_{l\delta}\hat{C}_{hp} \tag{32}$$

The necessary and sufficient condition for stability of rolling motion is given by the Routh-Hurwitz criteria[4]

$$B>0, \quad BC-AD>0, \quad D>0 \tag{33}$$

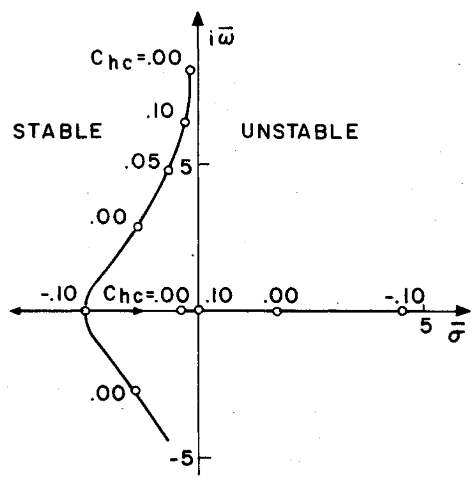

Fig. 3 Damping and frequency as functions of control hinge moment (for $C_{h\delta}=0.6$).

where $A>0$. If only D is negative, one real root of the characteristic equation is positive, and the motion is monotonically divergent. If only the discriminant $BC-AD$ is negative, the characteristic equation has two complex conjugate roots with a positive real part, and the motion has an oscillatory divergence. In any case, the number of unstable roots is given by the number of sign changes in the row A, B, $BC-AD$, D.

By considering the moments of inertia of a control surface, it can be shown that $A>0$. Normally, we also have $B>0$, since the dominant term of B is $-J_h\bar{C}_{lp}>0$.

The coefficient D is related directly to the control derivative defined in Eq. (23),

$$D=-\bar{C}_{lp}\frac{dC_{hc}}{d\delta_e}=-\bar{C}_{lp}(\Delta-3J_h\bar{p}_e^2) \tag{34}$$

When the parameter Δ is negative, i.e., when the effective aerodynamic damping in roll is destabilizing, we have $D<0$, and the rolling motion will diverge monotonically. When Δ is positive so that the missile has an effective aerodynamic damping, at small equilibrium roll rates we have $D>0$. At larger equilibrium roll rates the centrifugal term in Eq. (34) will prevail, thus $D<0$ and the motion will diverge. For a missile with a freely spinning tail, the transition from positive to negative values of D may occur at relatively low roll rates since the parameter Δ is relatively small.

The discriminant $BC-AD$ can have either sign at low equilibrium roll rates depending on the magnitude of the cross product of inertia J_{xz}. When the discriminant is negative, the motion has either monotonic or oscillatory divergence, according to the sign of D. At high roll rates the discriminant is negative due to the centrifugal term in $C_{h\delta}$, and as we then have $D<0$ the motion will diverge monotonically.

Figure 3 shows the roots of the characteristic equation as functions of the control hinge-moment coefficient C_{hc} for the case in which the motion has monotonic and oscillatory modes,

$$\lambda_1=\bar{\sigma}_1, \qquad \lambda_{2,3}=\bar{\sigma}\pm i\bar{\omega}$$

At $C_{hc}=0$ (free controls) the monotonic mode is stable, but as the hinge moment is increased somewhat the mode becomes divergent.

Figure 4 shows the damping of the monotonic mode as a function of the aerodynamic hinge-moment derivative $C_{h\delta}$ and the control hinge-moment coefficient C_{hc}. It appears that the damping $\bar{\sigma}$ varies appreciably with $C_{h\delta}$ and C_{hc} in the unstable region, but does not vary considerably in the stable region.

The effects of the inertia cross product J_{xz} of the control surfaces on the monotonic and oscillatory modes are shown in Figs. 5 and 6 for the control-free case ($C_{hc}=0$). Note that the damping property (i.e., the sign of $\bar{\sigma}$) of the monotonic mode is not affected by J_{xz}, as would be expected since the term D of the characteristic equation does not depend on J_{xz}. On the other hand, as $-J_{xz}$ increases the oscillatory mode may become unstable.

Numerical Solutions for Large Perturbations

To find how the rolling motion behaves for large initial perturbations, and how it is affected by missile pitching and yawing due to nonunison deflections of the control surfaces, numerical solutions of the full nonlinear equations of motion

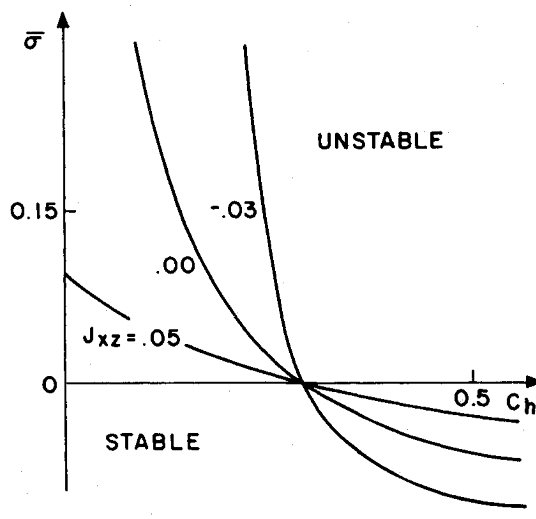

Fig. 5 Effects of control-surface inertial cross product on the monotonic mode (for $C_{hc}=0$).

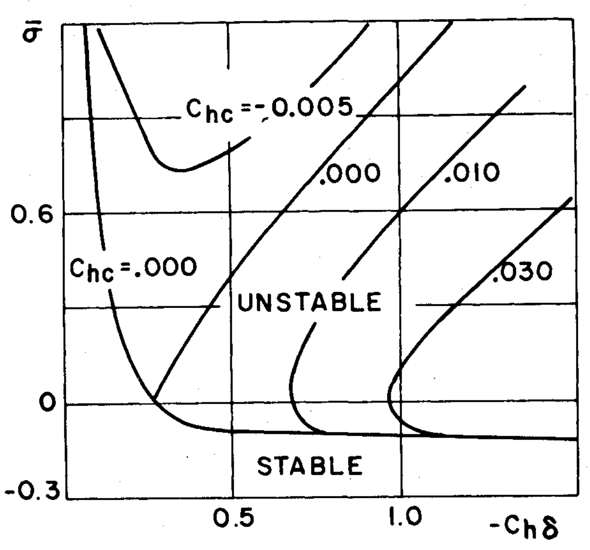

Fig. 4 Divergence or damping of the monotonic mode.

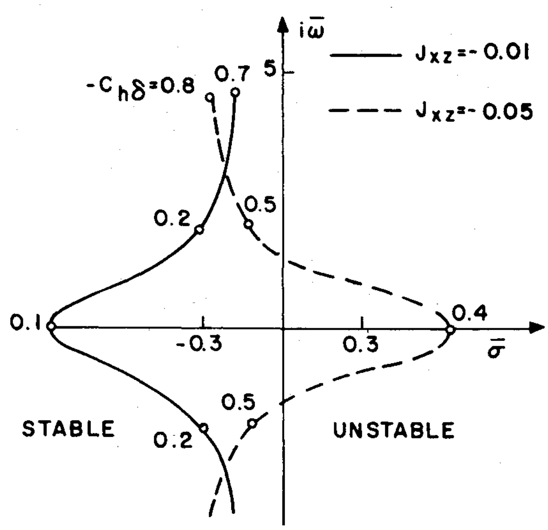

Fig. 6 Effects of control-surface inertia cross product and hinge-moment derivative on the oscillatory mode (for $C_{hc}=0$).

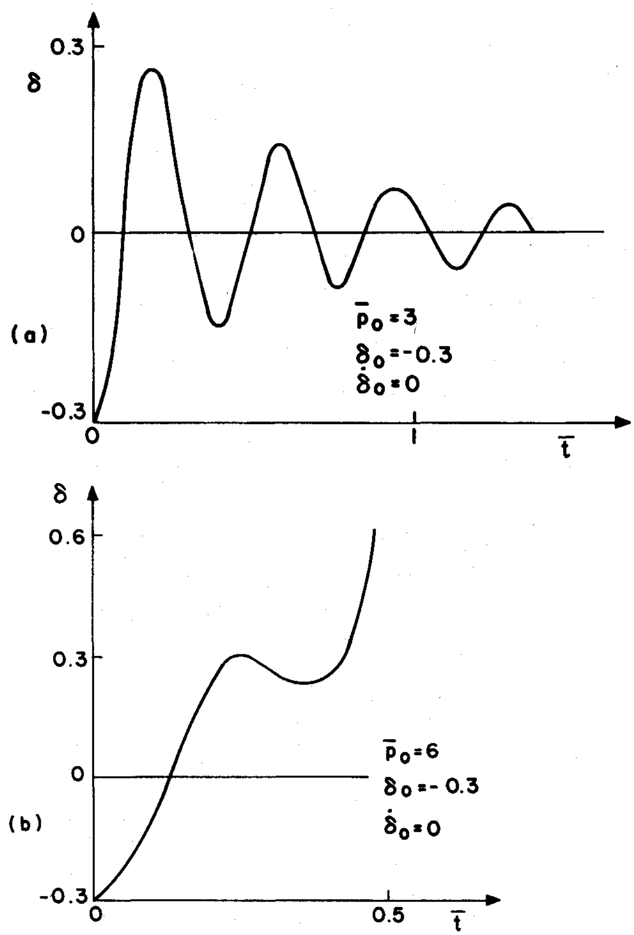

Fig. 7 Numerical solution for large perturbations in the control-free case.

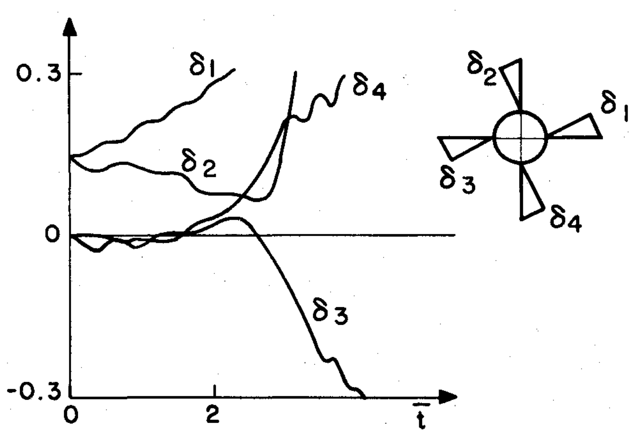

Fig. 8 Numerical solution for motion in six degrees of freedom (for $C_{hc} = 0$).

were computed for pure roll and for the general case of motion in six degrees of freedom with nonunison controls. In the general case, friction in the control hinges and tail bearing was included.

Figure 7 shows solutions for large deviations in pure roll from a stable equilibrium with free controls ($C_{hc} = 0$). The two examples differ in the initial rates of roll p_0, but have the same initial control-surface deflections. For the lower value of p_0 the perturbations are seen to damp out, but for the larger value of p_0 a divergent motion develops. The divergence is evidently due to the hinge moments of the centrifugal forces on the control surfaces.

Figure 8 shows a numerical solution of the general nonlinear equations of motion in six degrees of freedom with nonunison control-surface deflections. In this example, two consecutive control surfaces were deflected initially to produce rolling, pitching, and yawing moments, while the other two surfaces were set initially at a zero angle. In the ensuing motion all four deflection angles diverged strongly (Fig. 8). The solution indicated that in an incipient stage the variations of the deflection angles were due mainly to the aerodynamic hinge moments, but later the centrifugal forces on the control surfaces became dominant and led to divergence. The pitch and yaw angles of the missile remained small, as would be expected in view of the relative magnitudes of the inertial and aerodynamic moments in pitch and yaw as compared with those in roll.

Conclusion

An analytical study has been made of the rolling motion of a missile with a freely spinning tail and canard control surfaces actuated by constant hinge moments. It has been found that the rolling motion diverges in a wide range of parameters. The main factors generating the divergence are the loss of aerodynamic roll damping due to constant hinge moments, and the action of centrifugal forces on the control surfaces. Explicit conditions for the occurrence of diverging roll have been obtained by using linearized equations of motion for small deviations from equilibrium state. The divergent behavior of large deviations has been verified by numerical solutions of the full nonlinear equations of motion.

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